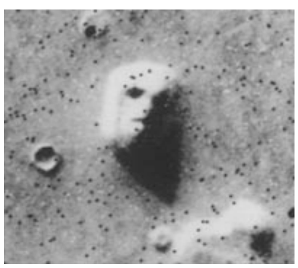
Measurements and data analysis - 1

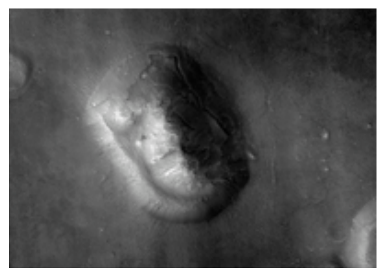
1.1 Scientific approach: observations, experiments and simulations

People observe what they expect to observe, until shown otherwise; our beliefs will affect our observations, and our subsequent actions. The purpose of the scientific method is to test a hypothesis, a proposed explanation about how things are, via repeatable experimental observations which can confirm or contradict the hypothesis, so as to fight this observer bias.

In July 1976, NASA's Viking 1 mission acquired images of the Cydonia region of Mars. The occurrence of an object with a seemingly human face caught the attention of public as evidence of a long-lost Martian civilization. More than twenty years later, higher resolution pictures were taken of the same area of Mars to end this controversy.



The Cydonia Region taken by the Viking 1 orbiter and released in July 1976.



High resolution *Mars Reconnaissance Orbiter* image of the "Face on Mars" taken in 2001.

Observations is data collected about objects or phenomena impossible to control or influence.
Experiments are repeatable and reproducible actions meant to validate a hypothesis or theory.
Simulations are simplified computer models of complex situation hard to study otherwise.

Assumptions of science:
Events follow patterns or result from causes that can be understood
Rules of nature are the same throughout the Universe, and always have been
Scientific reasoning is inductive: begins with specific observations and extends to generalizations
New evidence can disprove theories, but absolute proof is impossible

Skeptical thinking is a rational, free of bias, approach of accepting the implications of hard evidence. This is made of quantitative data (pieces of information), and usually, the more of it the better. We always look for independent confirmation.

1.2 scientific notation: numbers, exponents and units

The language of science is mathematics. The most common way of communicating data is to use numbers. However, in order to be sure that the transmission of data is complete and effective, we need to attach some more information and to stick to several conventions. There is not much point of sending a letter without attaching a properly formated address. The same situation appears when doing physics (or any science, for that purpose). Compare the following three sentences:

- the stone I hold is 1.2
- the stone I hold is 1.2 in weight
- the stone I hold is 1.2 kilograms in weight

Everything you report has to be in the following form (just an example):

mass = 1.2×10^3 grams *or* mass = 1200 grams *or* mass = 1.2 kg

When writing the number you can chose the floating point notation (123.45) or scientific notation (1.23 x 10^2 or 1.23e2) depending on your preference or convenience. It would be silly to write 0.00000000000123 instead of 1.23×10^{12} and it is much easier to write 0.1 instead of 1.00×10^{-1}, but ALWAYS think about and use the proper units.

The U.S. system of units and the Metric systems are both being used. Science and medicine tend to use predominantly the Metric system. You should be able to convert between the systems.

Unit conversions begin with equations which relate sizes of units, for example

$$1 \text{ yard} = 3 \text{ feet} \qquad (1)$$

is obviously correct, and so simple in appearance that the reader is likely to dismiss it as trivial.

But such physical equations relating measurables are significantly different from equations relating pure numbers. The above equation tells us that the measurement "1 yard" is equal (equivalent to) the measurement "3 feet." To write simply 1=3 would be incorrect (and absurd!). This equation relating measurements, is correct even though neither the numeric parts nor the unit parts are equal.

Equations relating measurements are manipulated by the ordinary rules of algebra, and the units are carried along according to the same rules. For example, if both sides of Eq. (1) are divided by 1 yard, the result is:

$$1 = \frac{3 \text{ feet}}{1 \text{ yard}} = 3 \frac{\text{feet}}{\text{yard}}$$

Therefore: 1 = 3 feet/yard. This last expression represents an identity relation for measurements. We call it a conversion factor. Suppose we had a measurement of 2.5 yards which we needed to express in feet. We would simply multiply by the appropriate conversion factor:

$$2.5 \text{ yards} = 2.5 \text{ yards } (3 \text{ feet/yard}) = 7.5 \text{ feet}$$

The unit "yard" has canceled by the rules of algebra, leaving a result in feet.

Some more examples:

1 mile/hour = (1 mile/hour) (5280 feet/mile) (1 hour/60 minutes) (1 minute/ 60 seconds) = 1.47 ft/s

$$\frac{(3 \times 10^{-8})(2.4 \times 10^{5})(6 \times 10^{2})}{(4 \times 10^{9})} = \frac{3 \times 2.4 \times 6}{4} \times \frac{10^{-8} \times 10^{5} \times 10^{2}}{10^{9}} =$$

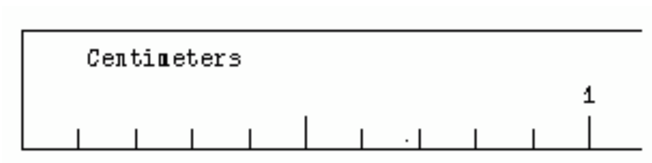
$$8.4 \times 10^{(-8+5+2-9)} = 8.4 \times 10^{-10}$$

PREFIX	SYMBOL	MEANING	EXPONENTIAL NOTATION
Exa	E	one billion billion	10^{18}
Peta	P	one million billion	10^{15}
Tera	T	one thousand billion	10^{12}
Giga	G	one billion	10^{9}
Mega	M	one million	10^{6}
Kilo*	k	one thousand	10^{3}
Hecto	h	one hundred	10^{2}
Deka	da	ten	10^{1}
Deci*	d	one tenth	10^{-1}
Centi*	c	one hundredth	10^{-2}
Milli*	m	one thousandth	10^{-3}
Micro	µ	one millionth	10^{-6}
Nano	n	one billionth	10^{-9}
Pico	p	one thousandth billionth	10^{-12}
Femto	f	one millionth billionth	10^{-15}
Atto	a	one billionth billionth	10^{-18}

1.3 significant figures: numbers, “exact” and “measured”

Some quantities we manipulate are measured. For that, a measuring instrument is used in a prescribed way in order to assign a number (value) to that quantity. For example, to measure a length we use a measuring instrument (a marked meter stick) in a specially prescribed manner (laying the stick off on the object to be measured) to determine the numerical value we call the "length." Other measurements may require more complicated instruments, or more complex techniques, but the principle is the same.

The meter stick example also illustrates the uncertainties common to all measurements. The smallest division marks on the stick represent millimeters. These are quite small, but with a little practice, most persons can estimate fractions of millimeter. Estimates smaller than l/4 millimeter are less reliable. This limits the accuracy of measurements made with a meter stick.



Suppose a small dust particle lies between two millimeter markings of the stick. We would like to specify the position of the particle. [This scale has its centimeter markings labeled 1, 2, 3, ... etc. Each centimeter is divided into 10 millimeters, which have marks, but are not labeled. Figure hows a magnified view of a one centimeter portion of the stick.] Figure 1 illustrates the particle's position as seen by the observer. We might specify the position by saying. "The particle lies between 6 and 7 mm on the scale." This statement specifies a range in which the particle is known to lie.

The position might also be specified by stating it "to the nearest millimeter." Since the particle is closer to 7 mm than it is to 6 mm, we would say, "The position is 7 millimeters, to the nearest millimeter." This statement also specifies a range in which the particle is found, but in this case the range is from 6.5 mm to 7.5 mm. This method of stating scale readings is very common in physical science. Usually, we simply state the reading as "0.7 cm" without explicitly specifying that the position was read to the nearest millimeter. Writing the value in this way implies the precision of the measurement, for if we had attempted to read the position to the nearest tenth of a millimeter we might have expressed the value as 0.78 cm.

This convention may be formulated as a rule:

Rule 1: Write values of physical measurements so that the last measured digit falls somewhere to the right of the decimal point. This may be done in either of two ways:

1. Use "scientific notation" (powers of 10).
2. Choose larger units of measurement.

Rule 2: The digit representing the smallest measured scale division must be explicitly written, even if it is a zero.

Rule 3 : Rounding. Parts (a) and (b) of this rule tell you how to discard insignificant digits, a process called truncating. Some people get fussy about a special case that occurs, when you must truncate (discard) just one digit that happens to be a "5". Should the final result be rounded "up" or "down"? Some advocate rounding (altering) the last digit retained so that it is even. Thus "3.785" would round to "3.78" while "2.755" would round to "2.76". When many numbers are combined, this rule serves to minimize the bias.

Example: A beam balance read to the nearest 1/10 gram reads exactly 30 grams. That implies that the reading is known to lie between 29.95 and 30.05 gm. The proper way to state this value is "30.0 gm." It is incorrect to simply write "30 gm" for that would imply a value in the larger range from 29.5 to 30.5 gm.

Example: The distance between two towns is measured to the nearest 10 meters and found to be 387,220 meters. To express this correctly we may write it in one of the following ways:
387.22 kilometers (rule 1, 2), or 3.8722×10^5 meters (rule 1) or 3.8722×10^7 centimeters.

1.4 measures of errors: mean value, the "±" notation

When repeated measurements of a quantity do not yield the same value, there may be some erratic influences on the measurement, or in the measuring process, larger than the smallest readable unit on the scale. The repeated measurements show a scatter of values. The scatter may have limited extent, so the measurement isn't completely uncertain. But we cannot predict (determine) exactly what the next measured value will be. Therefore these uncertainties (errors) are called **indeterminate**. Among many possible causes of indeterminate errors are:

1. Attempting to read an instrument scale to a very high precision. For example, trying to read a scale to the nearest 1/10 of its smallest division requires difficult estimation which may be highly unreliable.
2. Mechanical irregularities in the measuring instrument. For example, readings taken from a beam balance may be affected by friction and wear of its mechanical parts. Such effects may be reduced but never completely eliminated.
3. Uncontrolled (or unnoticed) outside influences on the apparatus.
4. Careless technique or observation by the experimenter.

Whatever the cause, indeterminate errors reveal themselves when repeated measurements give different values. A typical set of values for a measurement might look like this:

3.69 3.68 3.67 3.69 3.68 3.69 3.66 3.67

We assume that the measured quantity has just one precise value, independent of the measuring process, and that the variability of the recorded values is caused by imperfections of the instruments or procedure. We want to represent our knowledge, obtained from these measurements, as one "best" value.

We might represent this measurement by the **mean** (average) of the measured values. The mean is 3.67875 exactly. Recognizing that all of these digits are not meaningful, we round this off to 3.68, retaining the certain digits 3.6 and the first uncertain one, the 8. The value 3.68 is still somewhat uncertain. Nothing to the right of the 8 was certain enough to keep, but the 8 itself is a borderline case; it is not completely meaningless. We could be more precise and say that our value 3.68 is not likely to be "wrong" by more than ±0.02. The value ±0.02 is an estimate of the uncertainty in the value 3.68. Such results are written in standard form: 3.68 ± 0.02

Estimates or measures of uncertainty are called errors. In this case we have quoted a "maximum error" in the value. We will introduce other, better, kinds of error measures later.
Unfortunately data errors propagate through calculations, usually producing even worse error in the results.

Suppose that the number 3586.297 cm represents an experimental measurement, and we experimentally determined that its uncertainty was 0.2 cm. This number has seven digit accuracy and is expressed to three decimal places. The size of the uncertainty tells us that the digits 9 and 7 are superfluous, and carry no significant information. They express an amount smaller than the uncertainty. Such digits are called ***insignificant***. Insignificant digits can arise from mathematical calculations. Calculation devices such as electronic calculators display insignificant digits. They may also arise from reading an instrument scale beyond the inherent precision of the instrument.

The general rule is to discard all insignificant digits except the leading (first) one, and then to round off this uncertain digit. *Example:* A set of data has an average value of 3.645987 cm. The uncertainty is found to be 0.03 cm. Discard the 987. The first uncertain digit is 5. Round it down, to give the result 3.64

1.5 propagation of errors

An important feature of experimental data is that the errors combine and propagate through calculations to produce errors in the calculated results. The concept of significant figures is a crude and inadequate tool for dealing with this (we'll introduce better ways later). But it is instructive to consider how insignificant figures are generated in simple calculations. The operations of these examples are not ones you would normally do longhand.
Consider this multiplication example, in which uncertain digits are shown in bold italics.

395***4***
x 28***6***
____
23724
3163***2***
790***8***
____
11***39844***

Multiplying 3954 by the uncertain digit 6 gives a number in which every digit is uncertain. In the other sub multiplications, digits resulting from multiplication by the uncertain digit 4 must be counted as uncertain. Any column addition containing uncertain digits gives an uncertain result. Therefore only the first two digits of the answer are certain. The three is uncertain, and the remaining digits are completely uncertain.
Therefore this result should be rounded to 114. Notice that even though the multiplicand had four significant digits, the result has only three. This illustrates a rule:

Multiplication or division. Results of multiplication or division are rounded to the same number of significant digits as the least accurate data quantity.

The rule for addition is different because the decimal location of the first uncertain digit determines the location of the first uncertain digit in the sum.

Addition or subtraction. Find the data quantity whose last significant digit occupies the leftmost decimal place. This is the position of the last significant digit of the result.

Example.

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 0.52865
39.42
15.1
 0.02896
_________
55.07761
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which should be rounded to 55.1

1.6 Determinate errors

The preceding discussion of errors was devoted to indeterminate errors. This should not be taken to imply that determinate errors are not important. They are a constant source of trouble in experiments, and their detection and elimination may occupy a major portion of the experimenter's time.

While indeterminate errors show up clearly as scatter in data, determinate errors cannot be detected merely by a mathematical analysis of the data. A determinate error, if present, has constant magnitude and sign for all measurements of a particular quantity. Taking many measurements does not help either to detect or to eliminate the error.

Causes of determinate error are:
(1) Miscalibration of apparatus. This can be removed by checking the apparatus against a standard.
(2) Faulty observation. This is avoidable, and therefore should not be cited as a source of error in any well-performed experiment.
(3) Unnoticed outside influences. These are also avoidable, but may be difficult to discover.

In principle all determinate errors are avoidable, but their presence is not always obvious. The first hint of a determinate error may come when experimental results are found to be inconsistent with each other by amounts larger than predicted by the indeterminate-error analysis. Even when only one result is obtained, it may be inconsistent with results obtained by other experimenters or with previously established theory, indicating a possible determinate error.

In the elementary lab the problem usually shows up as a discrepancy between the experimental value and the "textbook" value. If the discrepancy is much larger than the indeterminate-error analysis predicts, it cannot be attributed to those error sources included in that analysis. One may suspect a blunder, and should then do whatever is necessary to identify it and conclusively show that it was the source of the trouble.

The cause may be an unrecognized determinate error. This should not be the end of the story, but rather the beginning of a thorough experimental search for the cause of the determinate error, and a demonstration that elimination of the suspected cause improves the result. Until this is done, any speculation about the cause of a "bad" result is only guesswork.

The physical or psychological causes of determinate error are, in principle, measurable. But if the cause was not suspected, the experimenter probably did not take the necessary measurements. One does not usually measure everything!

1.7 Quiz

Answer the following questions:

1. My friend from Europe says that his car is more fuel efficient than mine because it takes 7 liters of gas for each 100 km (that's how gas mileage is reported over there). Is he right? Assume that my car mpg is 35 miles/gallon. What my car's was 30 miles/gallon?

2. The work done on an electron of charge e = 1.6 x 10^{-19} C over a distance of d = 2.3 microns in an electric filed generate by the voltage 1.5 kV is given by *F x d = e x V.* Find the force *F* which act on the electron, from this formula. Express the answer in proper units and prefix. Example: mN, microN, etc.

3. A student wants to measure the mass of a dart by weighting a wooden block with and without the dart stuck in it, and then making the difference. Explain why is this not a good idea.